Summary of research work
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A major part of my research work is concerned with the following problem:

Although the concept of proof and the corresponding activity of proving are central to the doing of mathematics, the field of mathematics education has not yet theorized the teaching of proof and proving to support coherent learning experiences for students throughout the whole range of school grades.

Currently, much elementary mathematics teaching focuses on arithmetic concepts, calculations, and algorithms, and, then, as students enter secondary school, they are suddenly required to understand and write proofs. This creates a discontinuity in students’ experiences with proof in school that explains, at least in part, the fact that even advanced students demonstrate poor ability in proving. Also, the fact that proof is typically not a focus of elementary mathematics teaching deprives elementary students of opportunities to explore why “things work” in mathematics, which would provide them with a solid basis for conceptual understanding.

My research work is inspired by evidence from exemplary classroom instruction, which shows that it is possible for typical elementary school students to engage successfully in proving, and aims to address three broad research questions: (1) What are effective instructional practices for cultivating proof and proving in school mathematics (as early as the elementary grades), and in what ways do these practices support successful student learning trajectories in the domain of proof and proving? (2) What knowledge about proof provides high leverage for teachers’ work in implementing those practices in their classrooms? (3) How can this knowledge be effectively promoted in mathematics teacher education programs?

My work in addressing these questions required first the existence of a conceptualization of the meaning of proof that would be appropriate for even elementary school mathematics and that would support the study of instructional issues of proof. Given the scarcity of research on conceptual and instructional issues of proof, and on their connections, in school mathematics as early as the elementary grades, I undertook the task to develop a conceptualization of the meaning of proof in school mathematics that would serve the purposes, and constitute the foundation, of my research program. The process and outcomes of my efforts to develop this conceptualization are reported in a sequence of two articles (Stylianides, 2007a, b).

In the first article in the sequence (Stylianides, 2007a), I examined the characteristics of four major features of any given argument – foundation, formulation, representation, and social dimension – so that the argument could count as proof at the elementary school level. My examination was situated in an episode from a third-grade class, which presented a student’s argument that could potentially count as proof. In order to examine the extent to which this argument could count as proof (given its four major elements), I developed and applied two principles for conceptualizing the notion of proof in school mathematics: (1) The intellectual-honesty principle, which states that the notion of proof in school mathematics should be

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*I use the term proving to describe the activity associated with the search for a proof. Therefore, for me, the problem of clarifying the meanings of proof and proving in school mathematics reduces to the problem of clarifying the meaning of proof.*
conceptualized so that it is, at once, honest to mathematics as a discipline and honoring of students as mathematical learners; and (2) The *continuum principle*, which states that there should be continuity in how the notion of proof is conceptualized in different grade levels so that students’ experiences with proof in school have coherence. The two principles offered the basis for certain judgments about whether the particular argument in the episode could count as proof. Also, they supported more broadly ideas for a possible conceptualization of the notion of proof in school mathematics, a task that I undertook in the sequel of this article.

In the second article in the sequence (Stylianides, 2007b), I proposed a conceptualization of the meaning of proof in school mathematics and used classroom episodes from third grade to elaborate elements of this conceptualization and to illustrate its applicability even in the early elementary grades. Furthermore, I used the conceptualization of proof to develop a tool to analyze the classroom episodes and to examine aspects of the teachers’ role in managing their students’ proving activity. This analysis supported the development of a framework about instructional practices for cultivating proof and proving in school mathematics.

Next I present and make six general comments on the conceptualization of the meaning of proof that I proposed in Stylianides (2007b); full details can be found in the article. The conceptualization is presented in the form of a *definition of proof* that can be applied in the context of a classroom community at a given time:

Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:

1. It uses statements accepted by the classroom community (*set of accepted statements*) that are true and available without further justification;
2. It employs forms of reasoning (*modes of argumentation*) that are valid and known to, or within the conceptual reach of, the classroom community; and
3. It is communicated with forms of expression (*modes of argument representation*) that are appropriate and known to, or within the conceptual reach of, the classroom community. (Stylianides, 2007b, p. 291)

**Comment 1:** This definition is one among several possible approaches to conceptualizing the meaning of proof in school mathematics. Nevertheless, by presenting, in an explicit way, a possible way to conceptualize the meaning of proof in school mathematics, I hope to offer a specific context for discussions among researchers around this topic. To use Balacheff’s (2002/2004) words, “I do not expect every researcher to come on a same line, but [as a field] we may benefit from being able to witness our convergences and to turn our differences into research questions” (p. 1). Reid’s (2005) statement follows along similar lines: “[I]f [as a field] we can acknowledge that there is a problem, and discuss the characteristics of proof, we may be able to come to, if not agreement, then at least agreement on how we differ” (p. 7).

**Comment 2:** The definition describes a special class of arguments (those that qualify as proofs) without suggesting that other classes of arguments represent less valuable ways of knowing and doing mathematics. Also, although the definition focuses on what usually represents the end product of a mathematical exploration, this does not devalue the importance of the mathematical activities (e.g., pattern identification, conjecturing) and forms of reasoning (e.g., inductive reasoning) that often precede or support the development of proofs.
Comment 3: I consider the classroom community to consist primarily of the students. The teacher has a special membership status in this community as the representative of the discipline of mathematics (e.g., Yackel & Cobb, 1996) and as the person who has a special role to play in trying to connect students with broader mathematical knowledge.

Comment 4: In the definition there is an implicit reference to the community of professional mathematicians. The terms “true,” “valid,” and “appropriate” in the descriptions of the set of accepted statements, modes of argumentation, and modes of argument representation, respectively, should be understood in the context of what is typically agreed upon nowadays in the field of mathematics. Of course, this is not to say, for example, that the notion of valid modes of argumentation has universal meaning in the field of mathematics.

Comment 5: The definition describes the arguments that can qualify as proofs within a classroom community, but this does not mean that all individual learners who make up the community accept or (are able to) understand these proofs. As Lampert pointed out, the individual learners who make up the classroom community “go their separate ways with whatever knowledge they have acquired” (p. 310). Accordingly, when I talk, for example, about statements that are accepted by a classroom community, I do not wish to imply that each student in the community understands in the same way the elements of this set. Rather, I mean to call attention to the statements that can comfortably be assumed and used publicly without further justification. Finally, my focus on the classroom community does not suggest that I consider the individual to be less important. I agree with Lampert’s statement that “[t]eaching and learning need to take account both of what is accomplished by individuals and what is understood to be ‘true’ within the classroom discourse” (p. 310).

Comment 6: The definition satisfies the intellectual-honesty and continuum principles described in Stylianides (2007a).

References


