Research Paradigms on Teaching and Learning Proof

Nicolas Balacheff, short contribution (Providence Proof workshop, October 2007)

• **What theoretical assumptions underpin your research paradigm?**
  - *Knowledge is constructed*, and so is rationality (ref Piagetian constructivism) either as a result of the interaction with the environment (need to take decision, plan action) or with other human beings (socio-construction, collaborative learning).
  - *Language is a tool* consubstantial to the construction of rationality, or is required for it to be constructed (Habermas), but also language imposes its own constraints on the construction of argument (e.g. Duval, Ducroc about argumentation, semiotic laws in general).
  - *Mathematics as an activity*: the nature of the mathematical objects shapes the complexity of learning mathematical proof. Quasi-empirical nature of the learning of mathematics (need to revisit and understand Lakatos – bridging proving and knowing).
  - A didactical theory is a theory of validation. Proof is at the core of didactical processes.
  - *Situations* are key in the dynamic of proof, they are also one of the possible obstacles to its learning (problem of didactical engineering)

• **What are the big questions in your research paradigm that you would like to see answered (by yourself or by others)?**
  - What are the precursor of mathematical proof from an epistemological perspective (genetic epistemology)?
  - What are the optimal conditions for the learning-teaching of mathematical proof?
  - Need for a framework bridging proving and knowing.

• **What questions in your research paradigm have been answered by yourself or by others?**
  - A rather accurate view of the different types of proofs (arguments) and of their relation; their place in the genesis of mathematical proof is still to be understood, as well as their didactical “exploitation”.
    - need for a clear consensus, so that things can be taken for granted for a while and progress can be made without circular reconstructions.
  - Better links between argumentation and proof. However, the relation between explanation, verification and communication must be clarified and stabilized. They seem often supported in different papers by quasi-ideological positions. That all these aspects exist is almost a consensus. How they interact is an open question
  - There is a set of concepts [didactical contract, norms and customs] which acknowledges the institutional and social character of mathematical proof from a didactical perspective. As well as a good awareness of the double bind to which the teacher is submitted.
  - The cognitive unit across problem solving, proving and meaning making has been demonstrated by the Italian school, still to be articulated into a comprehensive model. The triplet (theorem, proof, theory) is at the core of this approach.
• What experimental techniques do you use to address your questions and what standards of evidence will you use to decide if a question is answered?
  - Didactical engineering
  - Mise en scène of students relations and interactions to elicit their proving processes (or rationale for their decisions, choice and validation)
  - Discourse analysis
  - Technics coming from the study of argumentation, like the Toulmin schema (without necessarily buying all the Toulmin theoretical framework)

• What particular sequence of experiments or analysis do you anticipate will lead to answers to the big questions you describe above?
  - Longitudinal studies across curriculum and evolution of the students through their school life. A real challenge.
  - Analysis of the micro-genesis of proof and validation
  - Analysis of classroom sequences following planed scenario and an explicit a priori modeling.

• Are there any researchers or schools of thought that you consider yourself to be aligned with?
  - The Italian school (Mariotti, Boero, Bartolini-Bussi)
  - Harel and Sowder to some extend (the psychological dimension to be questioned)
  - Brousseau theory of didactical situations (situations for validation)
  - Herbst model of the classroom as a societal-didactical phenomena/entity
  
  Actually it is more seeing where the contact can be established and further developed than an actual alignment. It may be a general position…

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**For a Taipei manifesto for research on teaching and learning mathematical proof**

Could we as a community...

• look for a common lexic and fix common definitions across our different languages, culture, institutions
• elicit the different *problématique* and their possible relationships
• elicit the theoretical commonalities and divergences
• comment on the different methodologies, their benefit and possible limits
• acknowledge accepted results
• stimulate duplication of pieces of work

⇒ turn objections and differences into research problems

Houlgate (FR) 1999 – Taipei (TW) 2002