My research (with Celia Hoyles) on the Proof Materials Project was concerned with ‘structural reasoning’ and how this can be developed in school students (we worked mainly with 11 - 16 year olds). Mathematical explanations of a conjecture or finding, ie explanations based on mathematical properties or relations, involve structural reasoning. Such reasoning is commonly based on empirical observations but, importantly, need not depend on the generation of systematic empirical data from which to induce a rule; it does however involve deduction so might be described as informal proof.

A key aspect of our work was to classify different kinds of proof task and in particular to find tasks that lent themselves to a structural approach, ie ones where students could short-circuit the empirical pattern-spotting approach common in UK schools.

A second key aspect was to work with successful teachers who were willing to take risks, in order to demonstrate that a diverse range of school students can engage in structural reasoning (a kind of existence proof).

Last, we explored ways of working in the classroom, in particular using a highly promising 4-phase approach (where students tackle a task on their own, then in groups, then as a whole class, then on their own again).

Our interest is in the nature and function of proof in school, so we have engaged with some of the ideas of Hanna, Bell, Balacheff, Dreyfus, de Villiers, Reid and Harel & Sowder, amongst others. For our classroom explorations we turned particularly to Hershkowitz, Cobb & Yackel and Sfard. Our perspective is social-constructivist, I suppose, with a firm belief that the commonly found limitations in students’ proof strategies are, to an important degree, the consequence of a lack of familiarity with or initiation into more mathematical ways of reasoning.

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