Our work in undergraduate mathematics education, with an emphasis on argumentation and the development of formal, rigorous ways of doing mathematics, is grounded in sociocultural theories (e.g., Lave & Wenger, 1994; Cobb & Yackel, 1996). Through classroom based research, our work strives to understand the processes by which learners develop increasingly sophisticated ways of participating in mathematics.

For example, in a recent paper on advanced mathematical thinking (Rasmussen, Zandieh, King, & Teppo, 2005), we depart from analyses that strive to characterize the nature of “advanced” thinking as a final state. Instead, we characterize advanced as a relative term that illuminates aspects of students’ progression and evolution of reasoning, in relation to their previous activity. For this reason we used the term advancing rather than advanced because we address the process of students’ total activity rather than just the final state. The term thinking is often used in describing mathematical growth to reflect a psychological point of view. While a focus on thinking often provides useful insights into inferred cognitive structures, it can result in neglecting the types of mathematical activities and ways of participating in these activities that foster and promote progressively sophisticated mathematical reasoning. Because we view learning as acts of participation in different mathematical practices, we intentionally use the term activity rather than thinking. Our use of the term activity, however, does not reflect a dichotomy between thinking and doing but rather intends to encompass both doing and thinking. We view the relationship between doing and thinking to be reflexive in nature, not dichotomous. As students engage in particular activities, they not only enact their understandings but also enlarge their thinking and ways of reasoning in the process.

In another paper (Zandieh & Rasmussen, 2007), we address how the activity of defining can serve the function for learners of constructing a new mathematical reality. At first glance it might seem relatively simple to indicate what is and is not a defining activity. For example, the most straightforward characterization of a defining activity would be “creating a definition.” However, our review of the literature and analysis of our case study data suggests that such a straightforward characterization misses the fact that formulating a definition, negotiating what one wants a definition to be (and why), and refining or revising a definition can occur as students are proving a statement, generating conjectures, creating examples, and trying out or “proving” a definition. We therefore found it necessary to include these other types of activity as part of defining when they involve formulating, negotiating, and revising a definition. As a further point of clarification, negotiating what we want the definition to be involves determining what words we want to use to state a definition as well as the deeper issue of clarifying what we want the concept to be.

One subtle difference between our work on defining and much of the existing literature regards the juxtaposition and/or comparison of concept image and concept definition. In most studies, the researchers make this juxtaposition. In comparison, in the student work we analyzed, it is the students who make this juxtaposition as a natural part of accomplishing their mathematical goals. Our work has also been influenced and has extended the instructional design theory of Realistic Mathematics Education (RME). For example, in our analysis of defining, we developed a means to frame the complexity of different types of defining activities in a way that adapted the RME heuristic of emergent models (Gravemeijer, 1999) to defining in a new way.
Our past work has focused on adapting the instructional design heuristics of Realistic Mathematics Education (RME) as a tool for creating courses that help students bridge the difficult transition from computationally based mathematical activity to formal, definition/proof based mathematical reasoning. Our current NSF funded project in linear algebra builds on this prior work. As the point of departure for this work we begin with Cobb and Yackel’s (1996) interpretive framework as a lens for analyzing the classroom discourse and student learning. This framework includes an interplay between individual and collective learning. The primary analysis in our past work has been to focus on the collective trajectory of student learning based on the contributions of individual students as they participate in classroom discourse. This point of view is particularly helpful for curriculum design and teacher decision making since both of these require choices appropriate to the learning of the class as a whole. However, not all students (and perhaps not even any one student) will have an individual learning trajectory that is identical to that of the group.

The importance of individual student learning within the reality of a teacher’s need to teach students in groups or as a whole class precipitates the need to more fully coordinate the individual and collective perspectives. Cobb and Yackel’s interpretive framework begins to address this issue. As we have worked within that framework on transitions between formal and informal reasoning, we have found that there are two related issues. As one tracks individual student learning, one may focus either on knowledge acquisition and cognitive structures or rather focus on a student’s participation in mathematical activities and discourse. In terms of collective learning trajectories there are classroom math practices that are specific to the particular mathematical content being studies and there are also practices such as defining, symbolizing, algorithmatizing and proving that are common across mathematical content areas but that are specific to the discipline of mathematics or have specific ways of being enacted within the mathematical community. Thus, we see the possibility and need to coordinate four different lenses, two that view the community as a whole, and two that focus on the individual. This new project seeks to build theory and methodology regarding these four viewpoints.

References