The guiding theoretical principles in our work are based on the notions that knowledge is socially constructed and that language is a significant psychological tool of development. Vygotsky (1962) maintained that “higher voluntary forms of human behavior have their roots in social interaction, in the individual’s participation in social behaviors that are mediated by speech” (Minick, 1996, p. 33), and that students’ development of self-regulatory thinking occurs through a process of internalizing events that originate on the social plane. Moreover, this process is indicated by the transition from social or public speech to inner, private speech. Advocating the mediating role of language in development, Halliday, reasoned that the "distinctive characteristic of human learning is that it is a process of making meaning - a semiotic process; and the prototypical form of human semiotic is language" (1993, p. 93).

From this sociocultural perspective, our work examines how tools such as language mediate undergraduate students’ thinking about proof as they participate in whole-class and small-group activity. In particular, using the methodology of design experiments (e.g., Kelly, 2003), we are currently looking at whole-class discourse in a one-year undergraduate mathematics classroom to try to understand how instructional scaffolding (Mercer, 1995) mediates students’ understanding of proof. The particular questions that guide this part of our work are:

- What are the forms of teacher and student utterances in classroom discourse on proof and how do these forms support or constrain students’ reasoning about proof?
- How are these forms connected to students’ passive or active participation in discourse, as inferred from the univocal or dialogic function of utterances?
- What is the evidence of development in students’ understanding of proof within the individual’s zone of proximal development and how is this evidence connected to whole-class and small group activity?

A subsequent part of this work will extend the analysis of whole-class discourse to examine how it impacts students’ thinking as they participate in small groups or reason about proofs individually. We are interested in possible patterns of transfer in the structure of whole class discourse to students’ small group discussions and the ways they reason privatively about proofs. Our premise is that students can acquire critical patterns of reasoning and argumentation through conversations facilitated by a more knowing other (e.g., the instructor) and that these patterns of discourse support their private understandings of proof. Alternatively, we see the absence of classroom conversations that prioritize reasoning and argumentation as having implications for students’ ability to reason about and construct proofs. Thus, the essence of our work is to understand the
mediating role of tools, particularly language, in the development of students’ understanding of proof.

We take a broad view of proof as a socially-constructed object whose purpose is to communicate the validity of a statement to a community based on criteria established by that community. That community might be children in elementary grades, or research mathematicians. Moreover, the degree to which a person needs to provide justifications for the ideas and representations used in a proof can be negotiated depending on the formality of the discussion within the community and its intended audience (Weber & Alcock, 2004; 2005). Even within the mathematics community, for example, proof is at the center of mathematical discussions that allow an argument to evolve in a dynamical manner (see, for example, Hanna, 1990; Lakatos, 1976), although that argument is ultimately subject to a formal chain of logic and specific premises agreed upon by that community. (We use Harel’s terminology of *mathematical proof* as referring to that distinct genre of proof unique to the mathematics community.)

References


