Proof and Generalization Research Paradigm
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My research focuses on addressing two central areas of inquiry related to proof:

1. What is the intersection between proving, generalizing, and transfer? When students conjecture and develop mathematical generalizations, how does the nature of their generalizing influence how they engage in proof activities? Moreover, how do students’ proving activities affect their generalizing, and how do these acts evolve over time and across domains?

2. How do classroom cultures, practices, and discourses affect the ways in which students engage in proof activities? In what ways do various aspects of instructional environments influence the development of students’ ideas about proof?

These questions are predicated upon a view of proof as determined by what constitutes ascertaining and persuading for the learner, following Harel & Sowder’s (1998) approach. Within this perspective, what constitutes a proof scheme is subjective to the learner’s experience and is shaped by the mathematical culture in which proving takes place. My approach to proof is also consistent with what Reid (2005) describes as the quasi-empirical view of proof.

Both of the above sets of questions are concerned with students’ growth and development – I am interested in studying how students’ proof abilities grow over time, how participation in particular ways of reasoning or particular classroom cultures influence developing ideas of proof, and how these interactions emerge and stabilize. I therefore view proof and proving as a continuum, in which students’ competencies grow as they gain facility with deductive reasoning and argumentation. Given the nature of these questions, constructivist teaching-experiment methodologies in the sense of Cobb & Steffe (1983) have been the appropriate method for informing my research agenda. I believe that theory must be informed by data, in this case the work of actual students in situ, and thus my thinking and resulting frameworks are data-driven and informed by grounded theory (Strauss & Corbin, 1990).

Results emanating from my research in the first area of inquiry are reported in three articles (Ellis 2007a; 2007b; in press). In the first article (Ellis 2007a) I focused on generalization and conjecture, documenting and categorizing the different ways in which middle-school students generalized and the different types of mathematical generalizations they produced in two instructional environments. I distinguished between students’ activity as they generalize, called generalizing actions, and students’ final statements of generalization, or reflection generalizations. By describing the major categories of generalizing actions and reflection generalizations that emerged from an analysis of students’ mathematical behavior, I located generalization within the learner’s perspective. This allowed for a move beyond casting generalization as an activity at which students either succeed or fail in order to allow the researcher to identify what students see as general; this view of generalization is consistent with my epistemology of proof.
In the second article (Ellis 2007b) I capitalized on the generalization framework and on Harel and Sowder’s (1998) proof schemes taxonomy in order to explore the interactions between generalizing and proving. This analysis led to the identification of four mechanisms for change that supported students’ engagement in increasingly sophisticated forms of algebraic reasoning: (a) iterative action/reflection cycles, (b) mathematical focus, (c) generalizations that promote deductive reasoning, and (d) influence of deductive reasoning on generalizing. Action/reflection cycles refer to engagement in particular generalizing actions, formalizing them as reflection generalizations, and then moving on to new generalizing actions. Although students’ initial generalizing actions and associated reflections were frequently limited or even incorrect, subsequent cycles built on previous attempts to develop more sophisticated generalizations. Mathematical focus addresses students’ focus, either individually or collectively, rather than the mathematical focus engineered by the teacher or a particular problem situation. The focus mechanism documents how students’ attention to particular mathematical properties over others can serve to either inhibit or promote their engagement in appropriate generalizing and proving. The final two mechanisms identify particular types of generalizing actions and reflection generalizations that supported students’ use of the transformational proof scheme, as well as documenting the ways in which students’ use of the transformational proof scheme enabled them to generalize more powerfully. Taken together, these mechanisms reveal a bi-directional relationship between generalizing and proving, showing how engagement in particular types of each activity can promote increased sophistication in the other.

The final article of the series (Ellis, in press) is concerned more deeply with students’ mathematical focus and its effect on proof and generalization. Examining middle-school students’ mathematical attention in two instructional environments in which they learned about linear function, I found that those who engaged in a particular type of quantitative reasoning were able to create powerful generalizations and provide appropriate justifications for them, which they were unable to do when attending to number patterns alone. This article more than the other two reflects the beginning of a move into the second area of inquiry outlined above, as it reports the influence of students’ engagement with different types of mathematical problems encountered in different instructional goals and environments. In the future I intend to continue exploring the second area of inquiry in more depth, particularly attending to the many ways in which micro-aspects of classroom cultures influence students’ engagement in proof activities.

References


