Research program on proof

Fundamental theoretical assumptions
In my research, I take a cognitive, process-oriented perspective on proving. My work is strongly influenced by information-processing theories of mind (Newell & Simon, 1972). (For a description of how information-processing theories of mind apply to mathematics education research, see Schoenfeld (1987) or my Weber (2006)). For tasks related to proving, including the construction, reading, and the evaluation of proof, I seek to understand the cognitive processes involved in performing the task competently, usually by inspecting the behaviour of ‘experts’ such as professional mathematicians, and comparing this with the processes used by students performing the same tasks.

Although my work borrows heavily on ideas and methodologies used from the information-processing school of psychology, I believe there are two important differences that distinguishes my work from this school. First, as Thompson (1982) notes, information-processing research often makes the assumption that all students are working within the same ‘problem space’ when performing a mathematical task, which to some extent implies that students are representing their task in similar ways. I disagree with this assumption and consider the ways that students and mathematicians represent the task of proving and the representations they use while proving as open and fundamental research questions. (For a difference between a sole focus on proving processes and research that looks both at proving processes and task representation, compare my early Weber (2001) and Weber and Alcock (2004)). Second, information-processing research tends to treat the solution or ‘goal state’ of a problem as independent of the individual. I also disagree with this assumption, and I argue that students’ goals for proving may vary widely. Students could be proving to gain knowledge, obtain a passing grade, convince themselves of an assertion, or some combination of all of these things. I have done little research on this issue, but consider the work of Herbst and Brach (2006) important in this regard.

Overarching goals of my research.
For proof-related tasks, specifically the construction, reading, evaluating, and learning from proofs, I would like to answer the following questions.

- What cognitive processes do experts (i.e., mathematicians) use when performing these tasks?
- What cognitive processes would we like students to use when performing these tasks? In particular, can we delineate a set of cognitive processes that are accessible to students, characteristic of appropriate and conceptual mathematical reasoning, and sufficient to perform these tasks competently?
- What cognitive processes do students use when performing these tasks? How do these processes account for students’ difficulties with proof?
- Can we design instruction that helps students develop and use a more sophisticated and effective processes when performing these tasks.

Research methods
I address these questions by performing task-based interviews with students or mathematicians or both. Usually, the data collected from this stage of the interview is analyzed using verbal protocol analysis (Ericsson & Simon, 1993; Chi, 1998) with the goal of describing individuals’ cognitive processes (e.g., Weber, 2001). This data is also used to hypothesize on what representations the individuals are using to complete the task and how they are representing their task. My goal here is to give an account that is consistent with and appears to explain the data. To corroborate these conjectures, I sometimes ask open-ended interview questions concerning these issues and use participants’ responses as confirming or disconfirming evidence for my conjectures (e.g., Weber & Alcock, 2004; Weber, in press).

Broader theoretical issues

I purposefully avoided broader epistemological questions, such as ‘what is a proof?’ and ‘what is its role in mathematics education?’ I do not have a coherent answer to this question and since this issue has been unresolved by mathematicians and philosophers of mathematics for over a century, I doubt that any mathematics educators can offer such an account either. However, I do not consider this question fundamental to my research.

Rather, I try to understand what students and mathematicians’ behaviour in terms of the goals that they are trying to satisfy. In this sense, my viewpoint is consistent with Herbst and Brach (2006) who ask what is proof to a student. They differ sharply from Harel and Sowder’s (2007) conception of proof in that I do not assume that students are trying to gain conviction for proving, unless they believe the purpose of proving is to gain conviction. (This is more a definitional agreement than a philosophical one. Obviously, I believe that proofs should be convincing and understanding what convinces students is important for understanding their behaviour and for the design of instruction).

I agree with Harel and Sowder’s (2007) contention that students should develop an understanding of proof that is consistent with that of the contemporary mathematical community. In this sense, I believe research on the mathematical community’s views of proof is valuable. I do not believe that questions about mathematical practice should be treated solely philosophically, but rather should be addressed empirically via the systematic examination of mathematicians, preferably while they are engaged in authentic mathematical tasks. I strongly concur with the viewpoint of Matthew Inglis that many of our assumptions about mathematicians’ practice are inaccurate and much can be gained by treating these issues empirically.

References
Herbst, P. and Brach, C. (2006). Proving and ‘doing proofs’ in high school geometry classes: What is ‘it’ that is going on for students and how do they make sense of it? Cognition and Instruction, 24, 73-122